Modelling statistical wave interferences over shear currents

Gal Akrish, Pieter Smit, Ad Reniers and ***Marcel Zijlema***

TU Delft

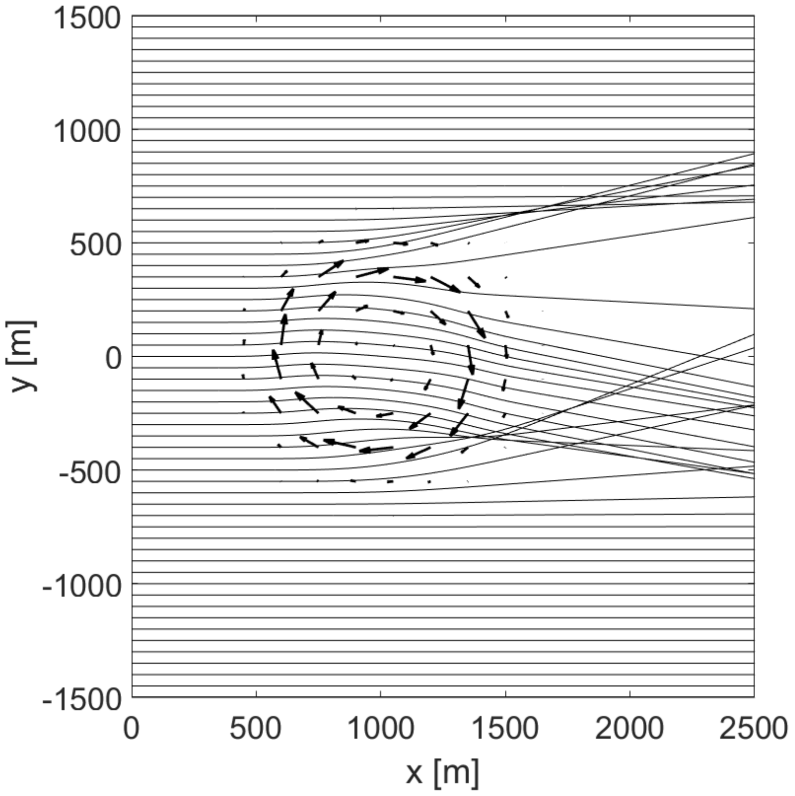
NWO

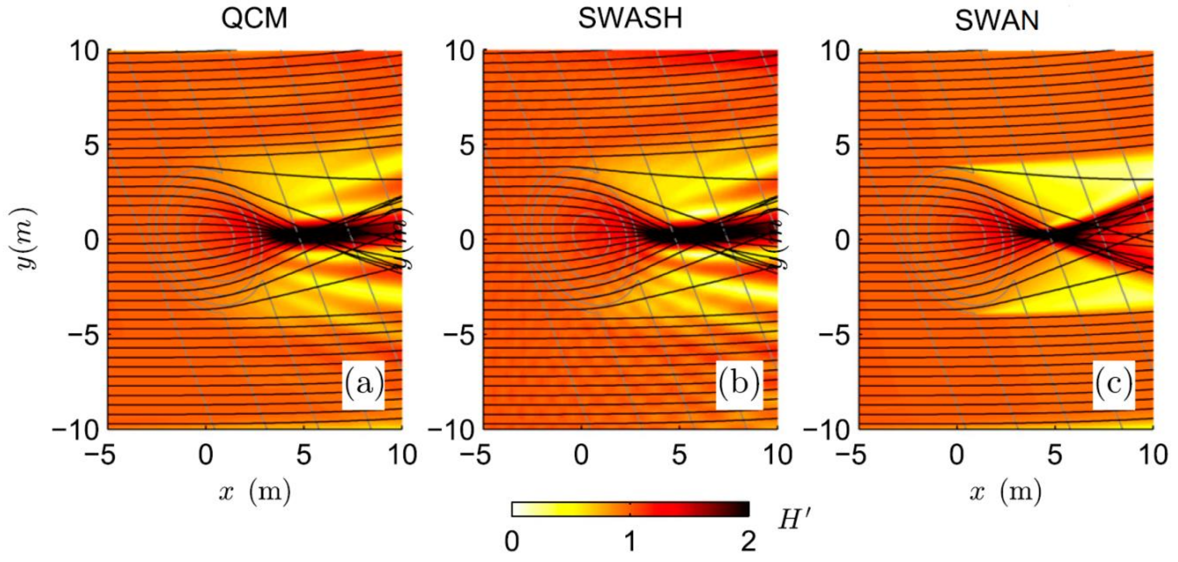
SOFAR

16th International Waves Workshop
Sheraton Hotel, Melbourne, Australia
13 November 2019

introduction and motivation

- recent studies by Smit, Janssen and Herbers (2013, 2015) demonstrate the relevance of interaction of waves with variable **bed topography** resulting in **coherent interferences**

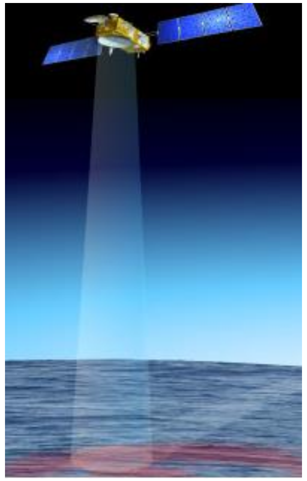




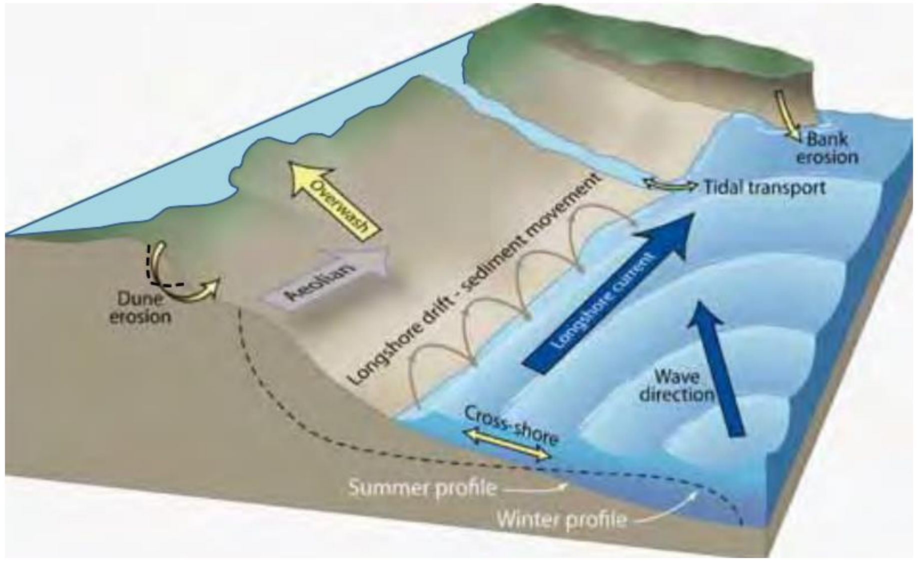
- in this talk we will present cases in which **inhomogeneous statistics** of waves over non-uniform **currents** become important

relevance and applications

- sediment transport
- wave driven currents
- extreme events in energetic focal regions
- measurements of small-scale currents



The Science Education through Earth Observation for High Schools (SEOS) Project (www.seos-project.eu)



Berman, G., 2011. Longshore Sediment Transport, Cape Cod, Massachusetts. Woods Hole Sea Grant Bulletin 46

National Oceanic and Atmospheric Administration (NOAA) (https://oceanservice.noaa.gov/facts/roguewaves.html)

background – current approach

- 3rd generation wave models to describe evolution of wind-generated waves in oceanic and coastal environment using **stochastic description** of the wave field (WAM, WW III and SWAN)

- assuming **Gaussian** and **quasi-homogeneous statistics** then wave field defined by the **variance** only → density spectrum of wave energy (or wave action)

$$N(\mathbf{k},\mathbf{x},t) \geq 0 \qquad \textit{(trace of spectrum tensor)}$$

- assuming **slowly varying medium** then the **radiative transport equation** (or the action balance equation) can be derived

$$\frac{\partial N}{\partial t} + \nabla_{\mathbf{x}} \cdot (\nabla_{\mathbf{k}} \omega N) - \nabla_{\mathbf{k}} \cdot (\nabla_{\mathbf{x}} \omega N) = \mathbf{S} \qquad \text{and} \qquad \omega(\mathbf{k},\mathbf{x}) = \sqrt{g|\mathbf{k}|\tanh(|\mathbf{k}|d)} + \mathbf{k}\cdot\mathbf{U}$$

background – limitations

- directional components are **statistically independent**
  - at deep water the wave field evolves slowly on scales of *O*(10 km – 100 km)

- in shallower water, however, wave components interact with medium
  - bathymetry and currents can vary rapidly in coastal regions, e.g. *O*(100 m – 1 km)

- they may become **correlated** and form **interference** pattern resulting in rapid variations of the mean statistics (in the near field)
  - effect more pronounced with narrow-band waves (e.g. swells)
  - interference patterns also occur in the presence of headlands, harbor entrances and coastal inlets …
  - … and around breakwaters, barriers etc. (**diffraction**) but also wave transformation over rip currents
  - **refraction** effects can be significant as well(!)

- also affect the far field statistics due to wave **focusing** and **defocusing**

objectives

- allowing for inhomogeneous statistics to be generated due to interaction of the wave field with **non-uniform currents**

- this study extends the results of Smit, Janssen and Herbers (2013, 2015) for cases of wave propagation over **rapidly varying bathymetry**

- implementation in the **Quasi-Coherent model** (QCM, Smit et al., 2015)

- two examples to demonstrate the capabilities of the extended model
  - ✓ swell field propagation over a narrow tidal jet (rip current)
  - ✓ swell waves that interact with an isolated vortex ring

generalization of the action density spectrum

- starting point is the **action variable** ψ representing a random and linear wave field over a varying medium
  - assumption: zero-mean, Gaussian and quasi-stationary
  - closely linked to the mean **action density**: $\langle|\psi|^2\rangle = m_0/\sigma$

- its statistics is defined completely by the correlation function

$$\Gamma(\mathbf{r},\mathbf{x},t) = \left\langle \psi\left(\mathbf{x}+\frac{\mathbf{r}}{2},t\right)\psi^*\left(\mathbf{x}-\frac{\mathbf{r}}{2},t\right)\right\rangle$$

- the **Wigner distribution** is derived from the Fourier transform

$$W(\mathbf{k},\mathbf{x},t) = \int \Gamma(\mathbf{r},\mathbf{x},t)\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, d\mathbf{r}$$

generalization of the action density spectrum (cont’d)

- since $\Gamma(\mathbf{r}) = \Gamma^*(-\mathbf{r})$, we have $W \in \mathbb{R}$ (not necessarily $W \in \mathbb{R}^+$)

- $W(\mathbf{k},\mathbf{x},t)$ provides a complete spectral description of the second order statistics of the wave field, including **cross correlation** contributions
  - these contributions correspond to **interferences** and can be negative

- the Wigner distribution $W(\mathbf{k},\mathbf{x},t)$ generalizes the concept of the action density spectrum $N(\mathbf{k},\mathbf{x},t)$

- via the equation of ψ, the Fourier transformed equation for $W(\mathbf{k},\mathbf{x},t)$ is

$$\frac{\partial W}{\partial t} = -\mathrm{i}\,\omega(\mathbf{k},\mathbf{x}) \exp\left[\mathrm{i}\,\overleftarrow{\nabla}_{\mathbf{x}} \cdot \frac{\overrightarrow{\nabla}_{\mathbf{k}}}{2} - \mathrm{i}\,\overleftarrow{\nabla}_{\mathbf{k}} \cdot \frac{\overrightarrow{\nabla}_{\mathbf{x}}}{2} \right] W + \text{c.c.}$$

evolution equation for inhomogeneous wave field

- the resulting equation is not feasible, therefore we follow the procedure of Smit and Janssen (2013) to carefully simplify

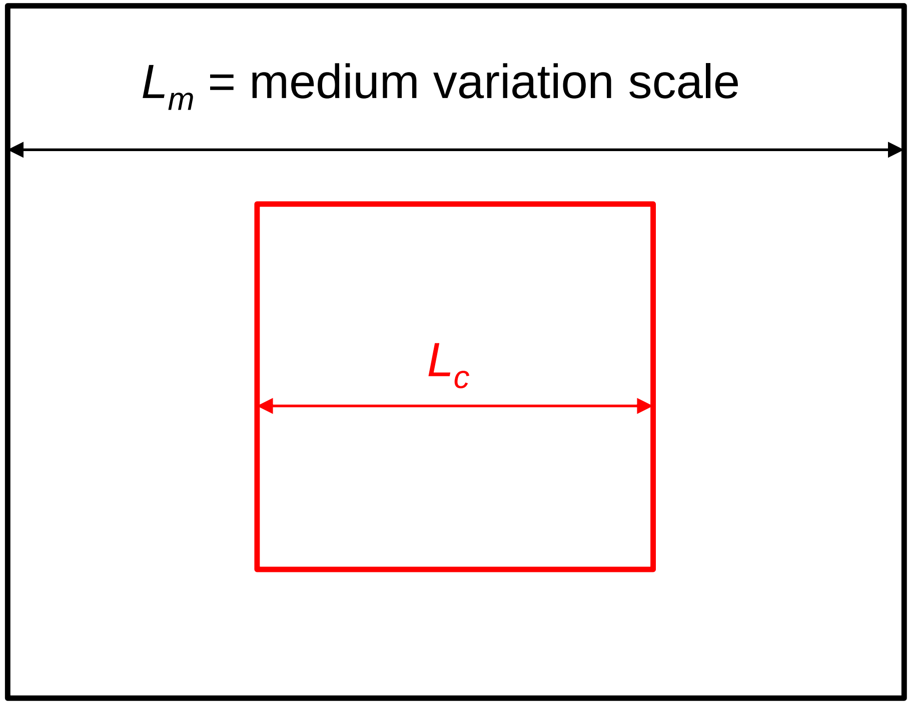
- introduce three scales (*L* is the characteristic wave length)
  - **medium** varies on scale $\boldsymbol{L_m = L/\varepsilon}$
  - **inhomogeneities** in the wave field due to medium variations vary on scale $\boldsymbol{L_w = L/\mu}$
  - the characteristic width of the spectrum is δ (=$\Delta k/k$) so that the **correlation** length scale is $\boldsymbol{L_c = L/\delta}$

- further assumptions are
  - the spatial variation of the interference structures is much larger than the characteristic wave length: $\mu \ll 1$
  - both narrow-band wave field ($\delta \ll 1$) and broad-band wave field ($\delta \sim 1$) can be considered

- relate the correlation scale to the medium variation scale: $\boldsymbol{\beta = L_c / L_m} = \varepsilon / \delta$

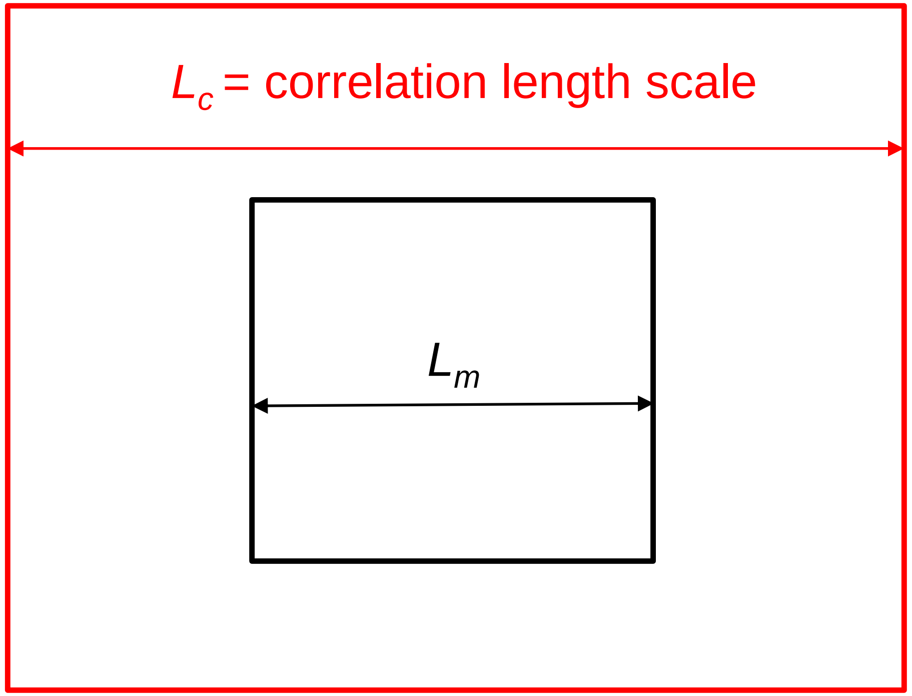
interpretation of β

$$\beta \ll 1$$



the wave field de-correlates over distances much shorter than medium variations

$$\beta \geq O(1)$$



significant changes in medium occur within coherent radius of the wave field

evolution equation for inhomogeneous wave field (cont’d)

- if $\beta \ll 1$ and $\mu \ll 1$, Taylor expansion reduces equation to **lowest order** to the radiative transport equation

$$\frac{\partial W}{\partial t} + \nabla_{\mathbf{k}}\omega \cdot \nabla_{\mathbf{x}} W - \nabla_{\mathbf{x}}\omega \cdot \nabla_{\mathbf{k}} W = 0$$

- if $\beta \geq O(1)$, a truncated expansion in β is not valid, however, since $\Gamma(\boldsymbol{r})$ has compact support, a Fourier integral yields proper equations

$$\boxed{\frac{\partial W}{\partial t} + \nabla_{\mathbf{k}}\omega \cdot \nabla_{\mathbf{x}} W = S_{\mathrm{QC}}}$$

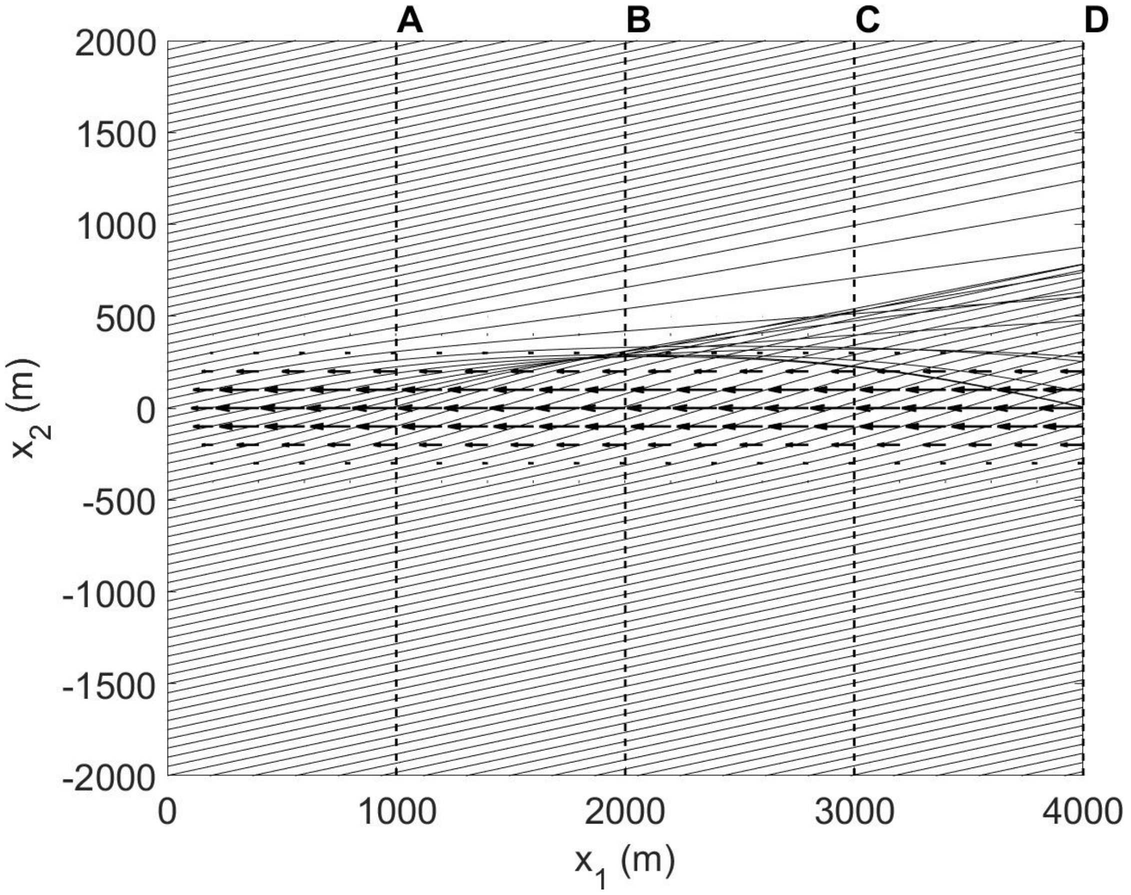
with scattering source term taking into account the statistical effects of **refraction** and **interferences** induced by medium variations

$$S_{\mathrm{QC}} = -\mathrm{i}\int_{\mathbf{q}} \omega(\mathbf{q},\mathbf{x},\mathbf{k})\left[1 - \frac{\mathrm{i}}{2}\overleftarrow{\nabla}_{\mathbf{k}} \cdot \overrightarrow{\nabla}_{\mathbf{x}}\right] W\left(\mathbf{k} - \frac{\mathbf{q}}{2}, \mathbf{x}, t\right) d\mathbf{q} + \mathrm{c.c.}$$

wave-current interaction examples

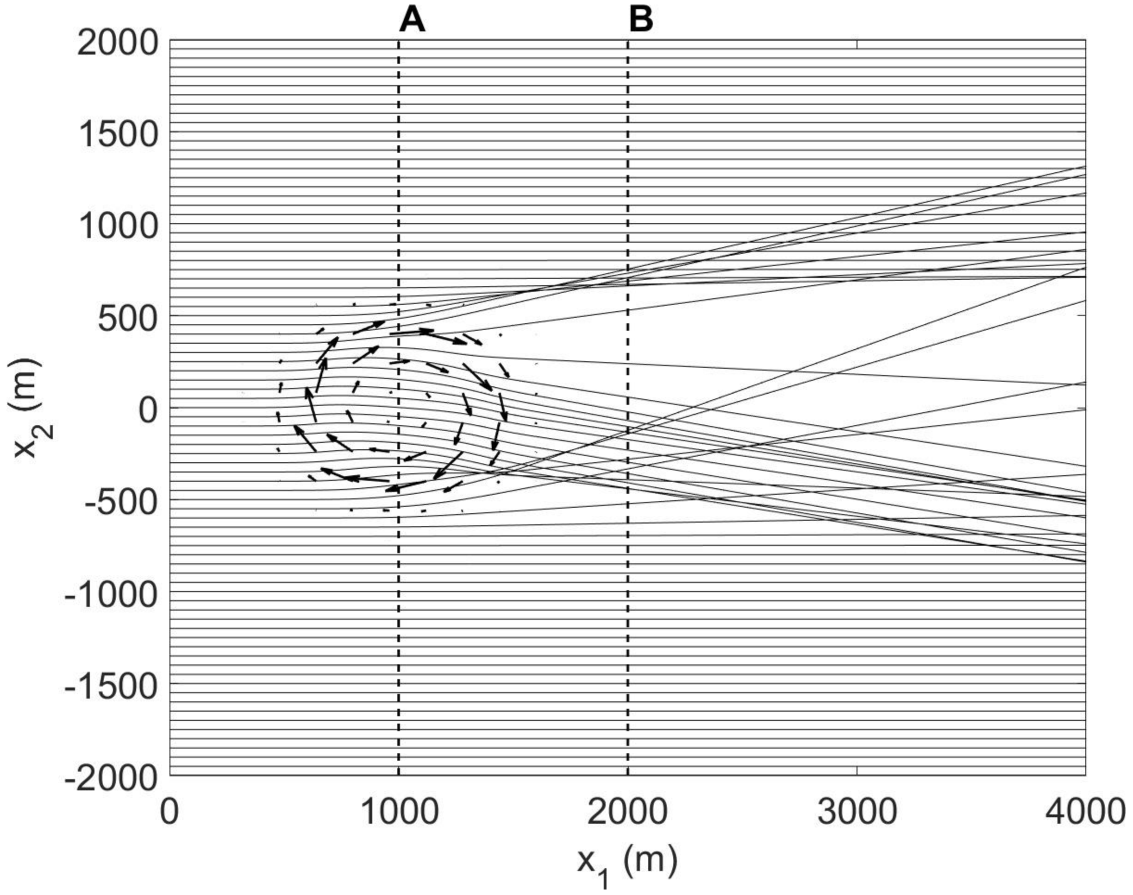
waves over jet-like current

$(d = 10\,\text{m},\ H_s = 1\,\text{m},\ T_0 = 20\,\text{s},\ \theta_0 = 15^o)$



waves over vortex ring

$(d = 10\,\text{m},\ H_s = 1\,\text{m},\ T_0 = 20\,\text{s},\ \theta_0 = 0^o)$



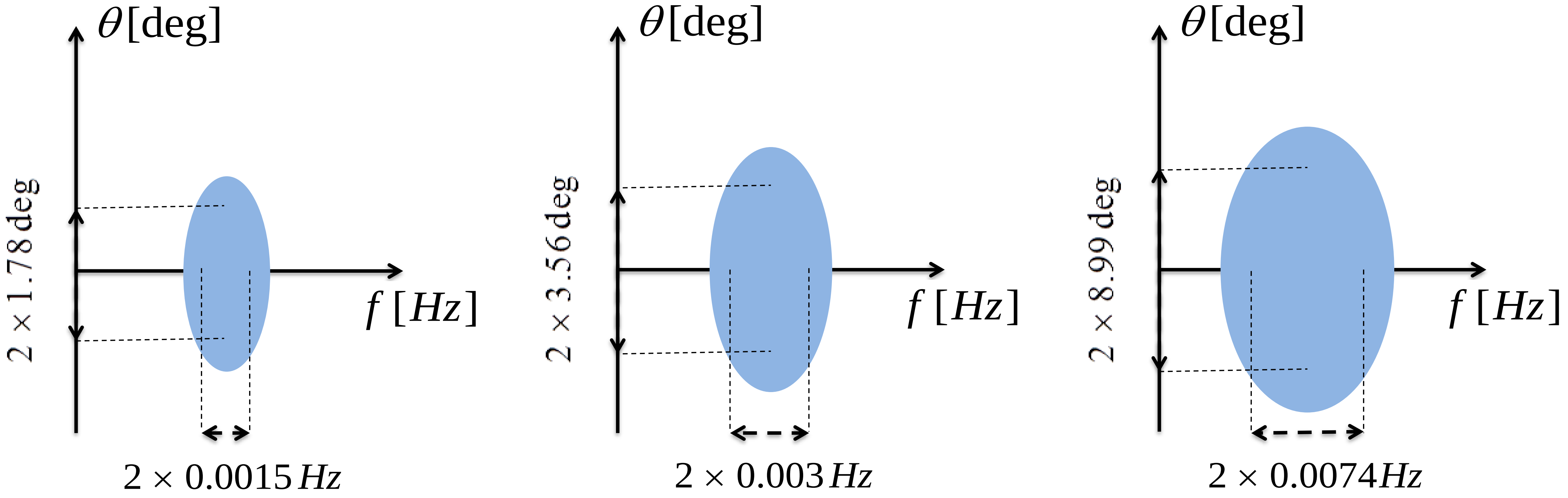
variance density at boundary

$$E(k_x,k_y)=E_0\exp\left[-\frac{1}{2S_d^{\,2}}\left(k_x-k_{x0}\right)^2-\frac{1}{2S_d^{\,2}}\left(k_y-k_{y0}\right)^2\right]$$

$S_d = 0.001\,\mathrm{m}^{-1}$

$S_d = 0.002\,\mathrm{m}^{-1}$

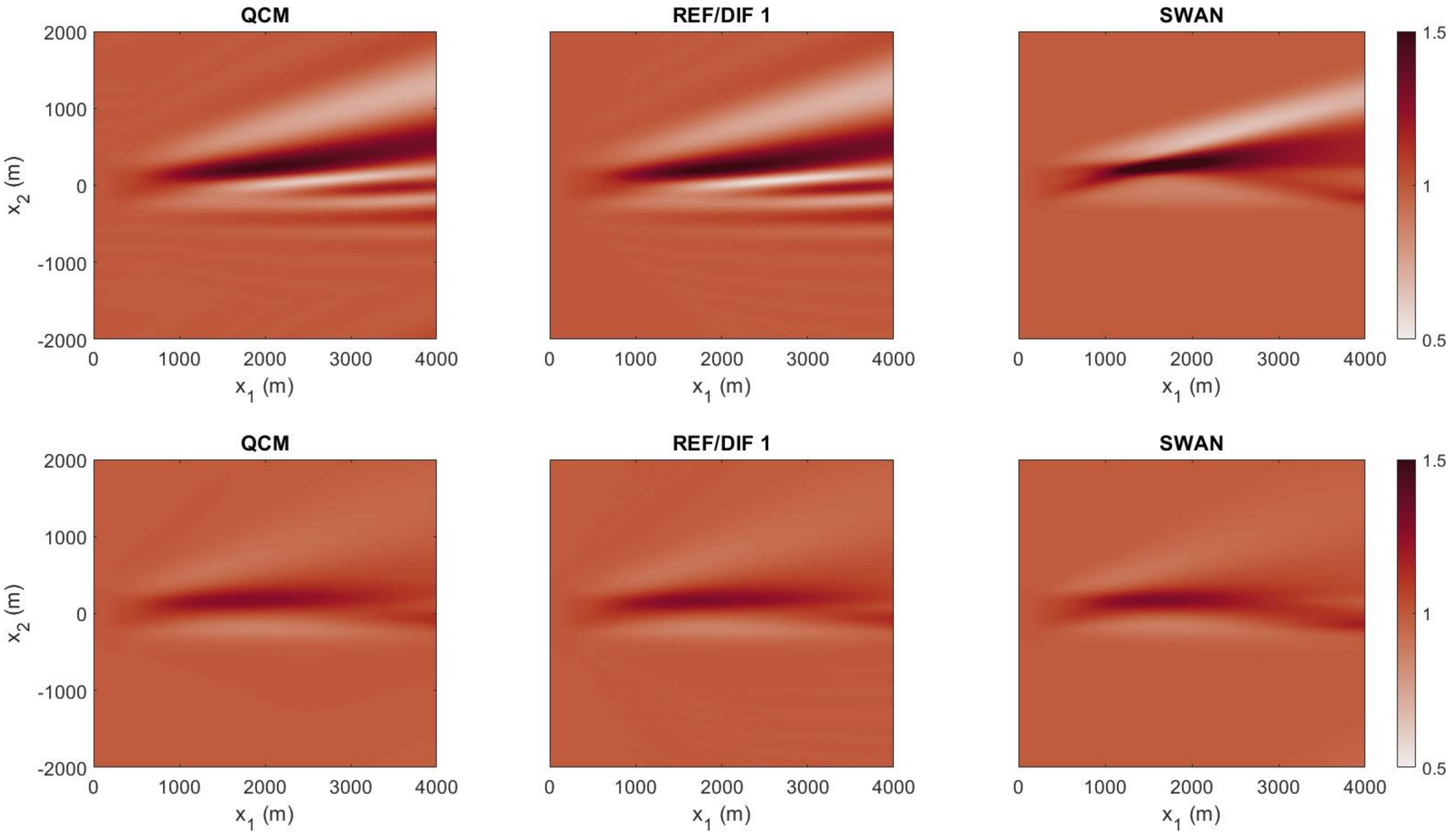
$S_d = 0.005\,\mathrm{m}^{-1}$



example: jet-like current

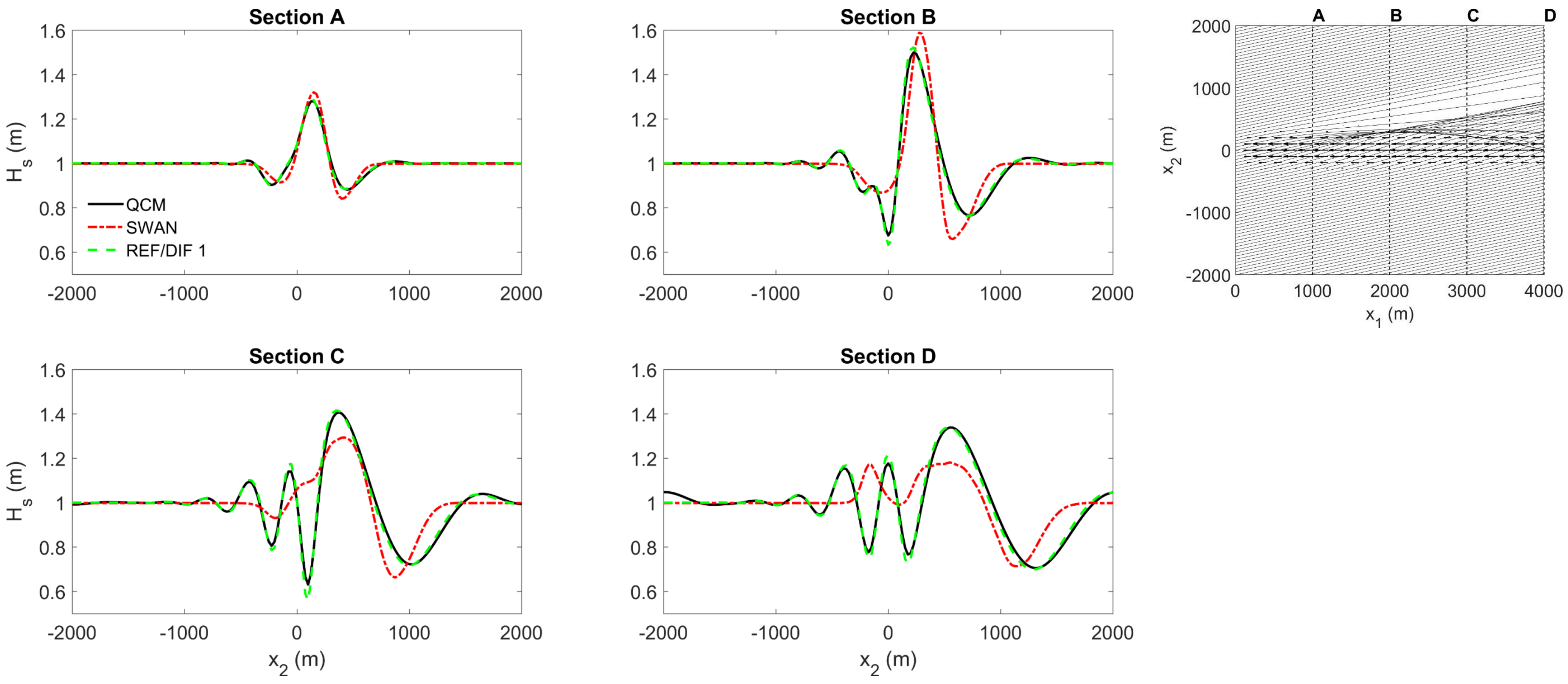
S_d = 0.001 m^{-1}
($\beta \sim 4$)

S_d = 0.005 m^{-1}
($\beta \sim 1$)

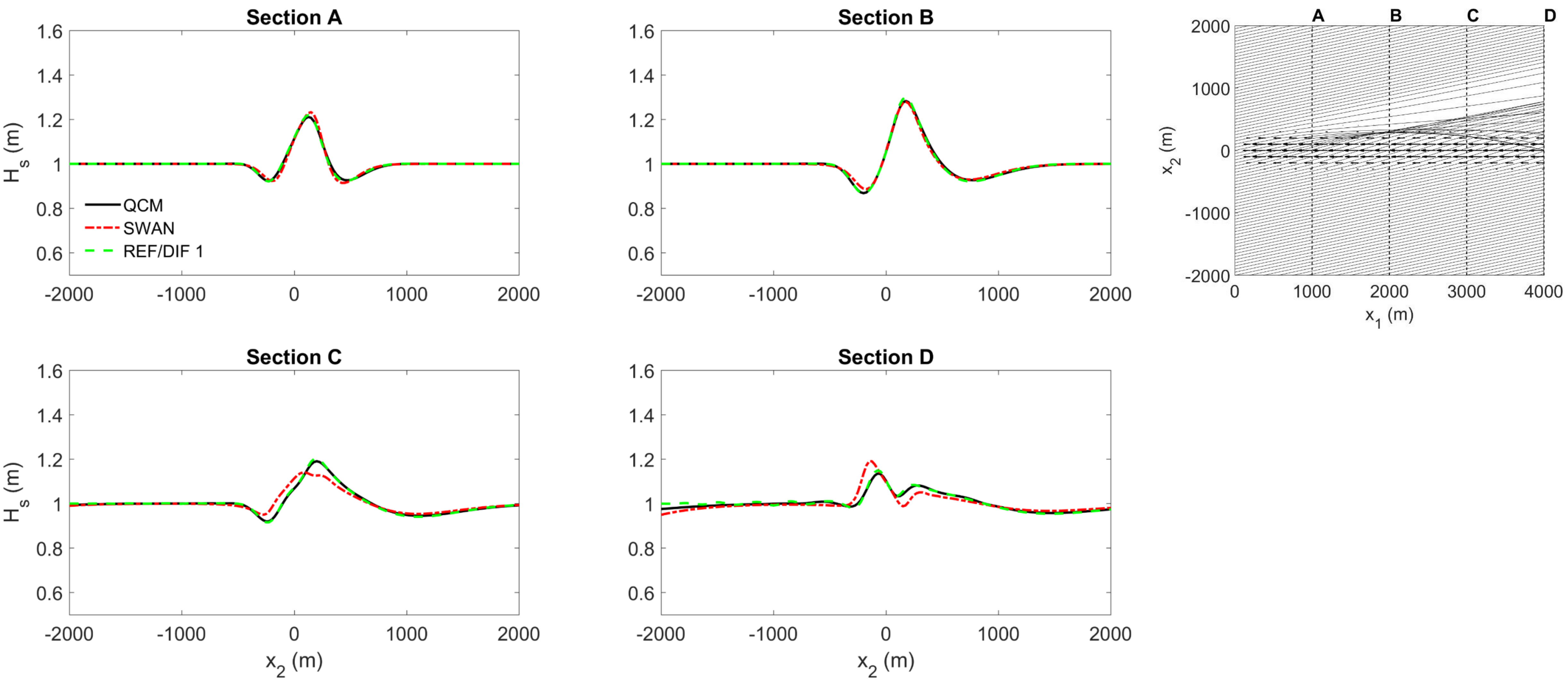


comparison between QCM, REF/DIF 1 and SWAN in terms of the spatial distribution of H_s

example: jet-like current (S_d = 0.001 m^{-1})



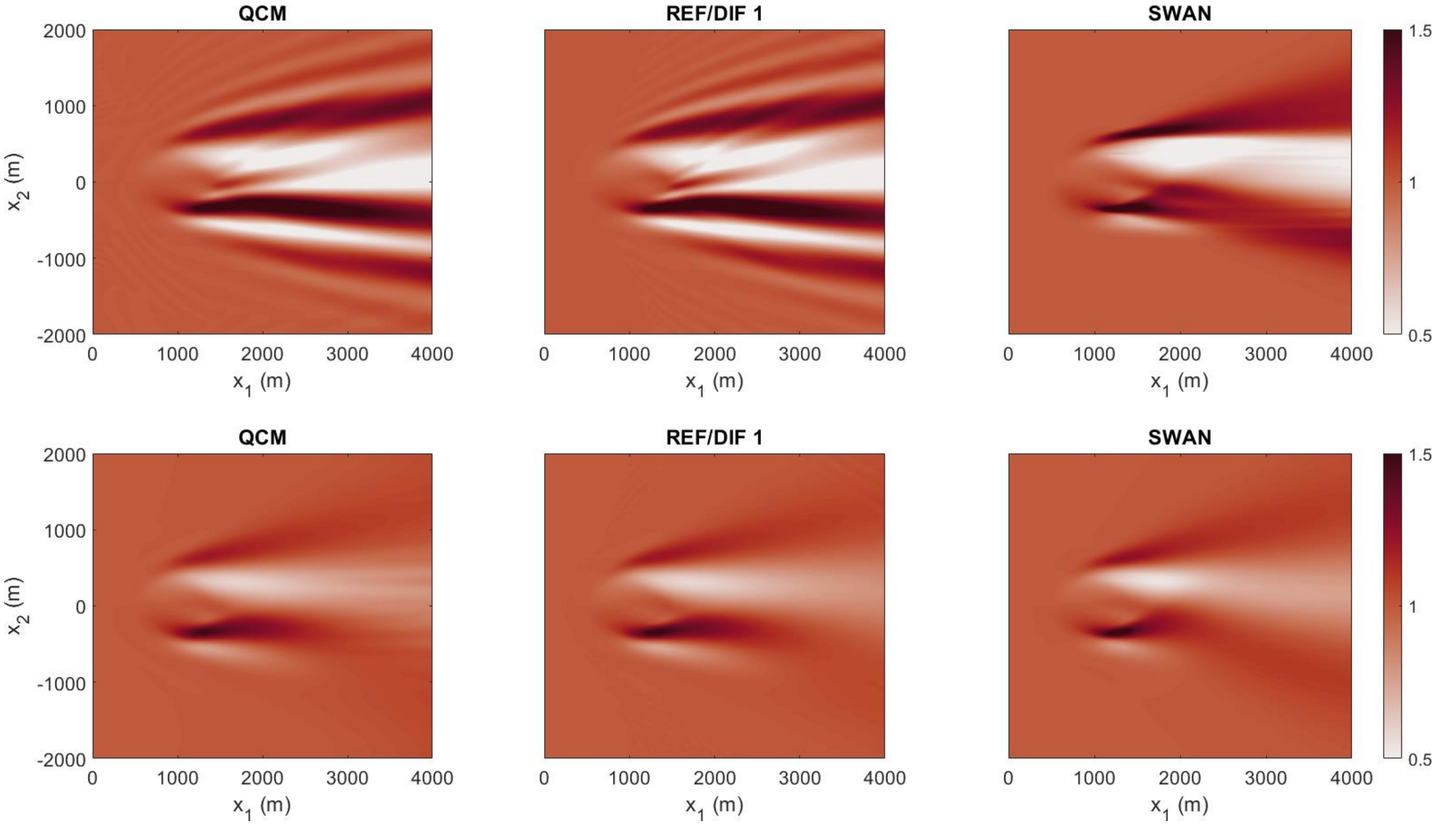
example: jet-like current (S_d = 0.005 m^{-1})



example: vortex ring

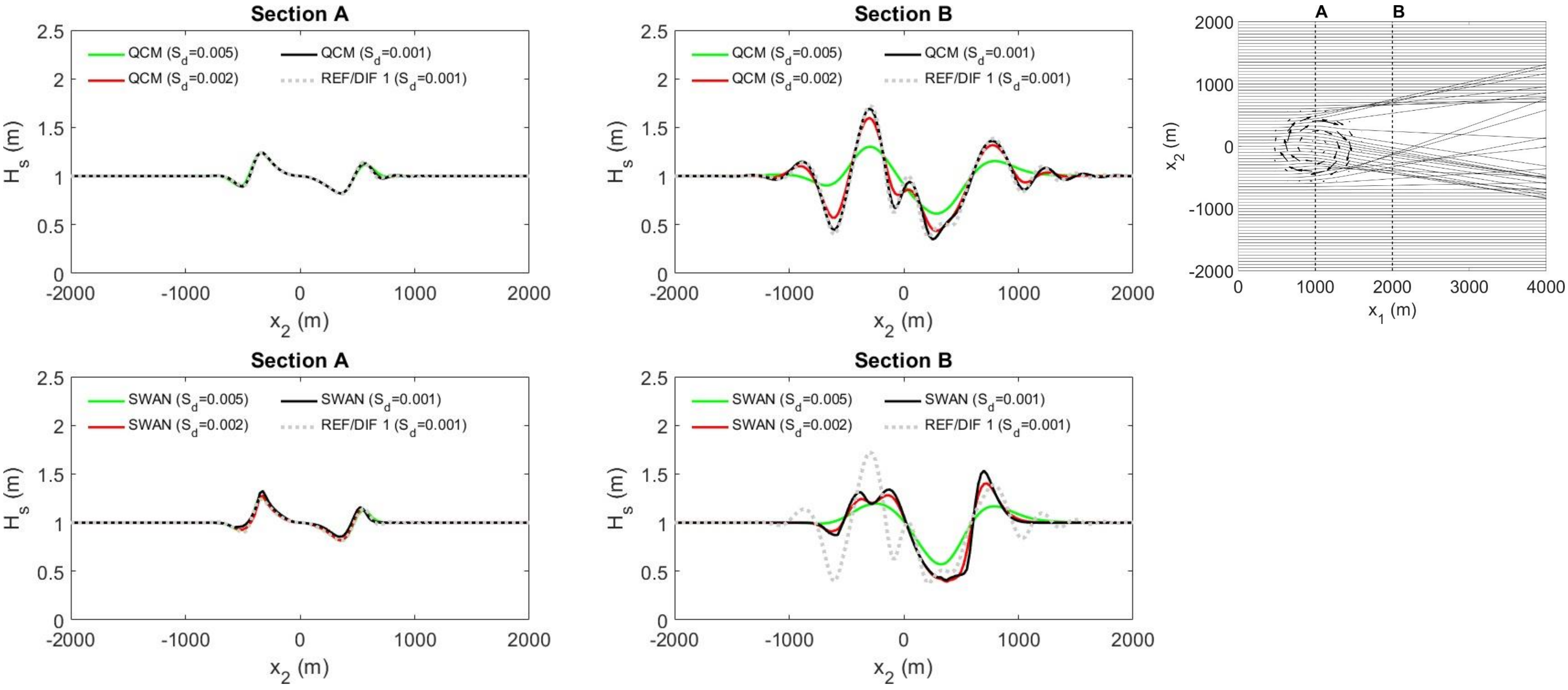
S_d = 0.001 m^{-1}
(β ~ 4)

S_d = 0.005 m^{-1}
(β ~ 1)



comparison between QCM, REF/DIF 1 and SWAN in terms of the spatial distribution of H_s

the validity of QCM



conclusions

- extension of the QCM for problems of wave-current interaction by taking into account the effect of wave interferences

- the Wigner distribution *W* is an extension of the action density spectrum *N* and provides a complete description of the second order statistics of the wave field

- an evolution equation for *W* is developed and is seen as a generalization of the conventional action balance equation by allowing the generation and propagation of cross correlation contributions

- generated cross correlations can alter the mean statistics significantly for cases where changes in currents occur over distances smaller than the typical scale of the correlation length

conclusions

- two synthetic test cases of wave-current interactions are provided
- a good agreement appears between the model results of the QCM and SWAN model until the crossing zones
- behind the crossing zones, in contrast to SWAN, the QCM captures the development of interference patterns due to correlation of crossing waves
- interference effects dramatically change the distribution of the significant wave height, also far away from the wave focusing area
- however, by increasing the spectral directional width, the agreement between the QCM and SWAN model extends even beyond the crossing zones